

AP Physics B Lab Manual

Welcome to the AP Physics B lab program. These labs have been designed to teach you good lab technique while giving you direct experience with the phenomena you're studying in the course. You'll perform at least 12 labs this year. The first couple of labs are a bit longer than the rest as they are designed to introduce you to the software and procedures we'll be using throughout the year.

The first lab uses real equipment which was provided in your student packet. With one possible exception, all the rest of the labs use simulations on our website. Hopefully you'll enjoy working with this virtual apparatus. It was all programmed by Nathan Pinney and Brian Vincent who were my assistants in 2006-07, 2007-09 respectively.

On the course web site there is a "Labs" button next to the "Mail" button which will take you to a page of links to all the labs. Feel free to visit the lab and mess around with the apparatus. You can't hurt it. You can also use it on your own to help you understand some of the phenomena we'll be studying.

You'll use this manual in three ways.

1. You'll use it as a lesson to work your way through before you do your first lab. There are a number of activities contained in it and you'll be asked to do many of them.
2. You'll use it as a reference when you're asked to do certain things in the lab. For example you'll be asked to find percentage errors. That's explained in section 1.4, and you'll learn (or review) that calculation when you study that section.
3. **You'll use it to learn how to use the Logger Pro software. Make sure that it's installed on any computer that you plan to use.** Your facilitator has a CD with the software on it. And you have a site license. You can even install it at home.

There's a lot of information in this document. Don't panic. You'll come to understand it better as the course progresses.

Will this be on the test?

This course is designed to help you learn a number of concepts and techniques that will help you understand and work with the physical world. The video and on-line lessons, the labs, and the homework work together to that end. You can expect to find test questions from all three of these.

You will need to thoroughly study this manual before you perform your first lab. It explains how we make and record measurements in the laboratory as well as how to use the Logger Pro software.

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Are you a PC or a Mac?

The lab apparatus and Logger Pro software will run on either a Windows or Mac computer. But the two sometimes handle things differently. Here's one to be on the lookout for.

Windows computers have at least two mouse buttons. – a left and a right, and sometimes a middle one. If you're told to **click** on something, this means to move your pointer over it and click with the **left button**.

We'll say **right click** when we mean to use the right one. Usually a right click brings up a menu of choices to select from.

Some **Macintosh** computers have just one button. It's equivalent to a regular (left) click. A **ctrl-click** (holding down the control key and clicking is equivalent to a **right click**.

Introduction

As you're learning in class, physics is about things that happen.

- A baseball is hit and travels along an arc or along the ground and the fielder has to get to the right place at the right time to catch it.
- Billiard balls collide at various angles and you can somehow predict which way they'll go.
- The tea I'm making right now will melt most of a glass of ice if I don't wait five minutes before I pour it in.
- The length of time between phone recharges depends on the screen brightness, amount of use, etc.
- The damaging effect of light depends on how much of various wavelengths are involved.

We'd describe these events in terms of distances, angles, times, temperatures, current and voltage, and wavelength and other quantities. In the lab we'll find out how these and other quantities are related by qualitative observations ("it's a very high fly ball"), measurement ("the tea is still cooling, I'll need less ice"), and mathematical description ("power \times time = energy, so maybe I'll get to talk more if I get a bigger battery.")

Let's look at some of the tools of the trade that help us make sure that our lab work that's up to code.

1. – Errors and Uncertainty

Scientific Error is not a bad thing. When we refer to **error** it's just another term for **uncertainty**.

The measurements you make in the laboratory should be made as accurately as possible. The numbers you record, with appropriate units, express the amounts of your measurements as well as their uncertainties (error). (We often use the terms uncertainty and error interchangeably.) This lab manual will introduce you to the techniques we'll use to accomplish this. The methods of error analysis and even the definition of key terms found in the literature vary considerably. In future courses you may be introduced to more sophisticated error analysis.

1.1 – Types of Error

Our measurements are limited in two ways – **instrument limitations** and random **uncertainties** or **random errors**.

Instrument limitations: Our measuring devices have limits to how finely they can divide up a quantity. The finest graduation on a meter stick is a millimeter. We can use it to measure to within some fraction of a millimeter, but not beyond. We would say that 1 millimeter is the **resolution** of a standard meter stick. If we removed the millimeter marks, leaving centimeter graduations we would decrease its resolution to one centimeter. Thus we would reduce its resolution by a factor of ten.

Random (indeterminate) errors: When you measure the length of an object with a meter stick you have to judge when the object and meter stick are aligned properly, which points on the meter stick align with each end of the object, etc. If you made the same measurement several times you'd get slightly different results, ranging by some amount either side of some central value. This type of random variation is inherent in any measurement, but the average of many such measurements would be a good indication of the "correct" value.

What about human error? This is not actually a scientific term. The two types of limitations mentioned above are normal parts of any scientific measurement and we can deal with them in standard ways as you'll see. But "I didn't realize that I was using a yardstick and not a meter stick" is just a mistake. We'd deal with it by repeating the experiment with the proper tool. So just eliminate that term "human error" from your scientific vocabulary.

1.2 – Precision and Accuracy

To its extreme embarrassment, precision's definition varies according to the context of its use. In physics we generally use the term in two ways.

1. **Precision** is the degree of agreement among several measurements of the same quantity; the closeness of repeated measurements to the same value; the reliability or reproducibility of a measurement.

Note that precision is **unrelated to the correctness of a measurement**. If you shoot at a target several times and all the arrows are close together you're a precise shooter. If the arrows are actually in the wrong target, you're a precise but inaccurate shooter.

You may see precision expressed like this: $2.04 \pm .05$ m. This would indicate that your average value for several measurements was 2.04 and that the measurements were spread over a range between 1.99 m and 2.09 m. So you feel confident that any subsequent measurement would fall into that range. A less precise measurement of the same object might be written as $2.04 \pm .1$ m. And $2.04 \pm .02$ m would be more precise.

2. **Precision** is the measure of how exactly a measurement is made; the number of significant digits to which it can be measured reliably.

According to a plastic ruler a nickel is about 2.10 cm in diameter. If I used a device called a Vernier caliper I can measure more precisely where I might find the diameter to be 2.096 cm. The extra digit indicates the greater precision of the instrument being used.

In our work the term usually refers to the second definition. We also use the term **resolution** when using this definition. A plastic school protractor might measure angles to a few tenths of a degree, while a sophisticated scientific tool might measure to thousandths of a degree. Similarly a photo taken by a satellite passing over Mars might have a resolution of 10 meters. This means that the light from a $10\text{m} \times 10\text{m}$ area on Mars illuminates just one pixel on the digital camera's light-gathering chip. So if the light from that area is reddish on average, the pixel will be reddish. A similar effect is found with your eyes. As you approach a distant object the image spreads over a larger area on your retina, increasing the resolution of the image, letting you **resolve** smaller objects.

Ex. "I see beach." "Now I see sand." "Now I see that the sand is multicolor and jagged."

Closely related to precision is accuracy. **Accuracy** is the degree of closeness of a measured or calculated quantity to its actual (true) value; the extent to which the results of a calculation or the readings of an instrument approach the true values of the calculated or measured quantities.

The results of calculations or a measurement can be accurate but not precise, precise but not accurate, neither, or both.

This may be better understood by an analogy. Consider several attempts by a marksman to hit a bull's eye. If the bullets all hit in a tight pattern we'd say that the shooting is very precise. This would be true even if the tight cluster is far from the bull's eye.

If that tight cluster was centered on the bull's eye we'd say that the shooting was both precise (definition 1) and accurate.

If the cluster was not so tight, but still centered on the bull's eye, we would say that the shooting was accurate, but not precise.

We won't be doing any shooting in the lab, but we will be making multiple measurements of quantities. If our repeated measurement of a quantity are nearly the same (precise) and approximately equal to the "correct" value (accurate), we'd say that our measurements are both precise and accurate. A result is called **valid** if it is both **accurate** and **precise**.

Incidentally, the shooter whose bullets hit in a tight cluster away from the bull's eye would be said to have a **systematic error** in his shooting. What would the judges in that event have to say about this? "Sorry buddy, we don't forgive the 'human errors' of the choice of an inadequate weapon, or the improper adjustment or use of the gun sight." You lose.

Systematic errors are not related to uncertainty. They indicate that you've done something incorrectly and need to correct the source of the error and retake your data. For example, if you were weighing liquids in a beaker and forgot to subtract the weight of the beaker all your values would be offset by an amount equal to the weight of the beaker.

1.3 – Uncertainty of Measurement

All scientific measurements are made by comparison to some accepted standard. The length of the spine of a book is compared to markings on a standard meter stick; the time for a ball to fall some distance is compared to the seconds ticked off on a stopwatch, etc.

When I asked a visitor to my house how tall he was he responded "2 meters, 4 centimeters." Was this tall Scandinavian exactly 2.04 m tall? Who knows? He was just really tall and that's all that we were curious about.

But as a scientific measurement 2.04 meters means something very specific. If we measured the distance that a rock had fallen to be 2.04 m, this value would mean that we know it fell some distance between 2.00 m and 2.10 m and that the 4 cm is our best guess of the final digit. We will always use this system when we take data.

You should always measure as many digits as you can with certainty and then estimate one more digit.

For a typical length measurement in the lab we can measure to a much higher resolution. A meter stick has millimeter markings on it. Let's measure the length of the spine of a book with the section of meter stick shown below. Our measured length would be the difference in the right and left measurements. The left measurement is between 6.1 cm and 6.2 cm. We need to estimate one more digit. Let's say 6.14 cm. Our uncertainty is due to our inability to **resolve** tenths of millimeters. It's also compounded by the curve of the spine and the separation between the book and the ruler. The other end seems to be between 12.7 and 12.8. We'll estimate 12.72 cm.

We'll record the book's length as $(12.72 - 6.14) \text{ cm} = 6.58 \text{ cm}$. Note that our answer gives the magnitude of our measurement as well as an indication of the uncertainty of our measurement. We are confident to within .1 cm.

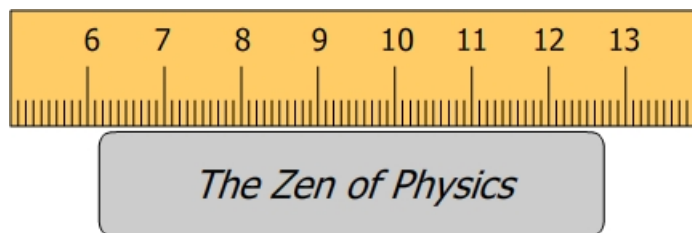


Figure 1

There are many more sophisticated ways of expressing and calculating uncertainty. You may encounter them if you continue in your scientific studies. Let's quickly look at one that you may be familiar with before we proceed. (We won't use this method.)

Returning to Lars' height, we could go one step further and state how certain we are about that last digit. As you know, measuring a person's height is pretty uncertain given the squishiness of hair and the difficulty of getting the mark on the wall just right. So on a good day, you'd be happy with getting within a centimeter. So you might give Lars' height as $2.04 \pm .01 \text{ m}$. (**value \pm uncertainty**) The $\pm .01$ indicates that you are confident to within .01 m

Beyond this lies statistical analysis using standard deviations and partial derivatives for the calculus students among you. Again, we won't use these methods.

Significant Digits (Figures)

In the two examples above the uncertainty of our measurements (hundredths (2.04 m) and ten-thousandths of a meter (6.58 cm)) has been clearly indicated by the digits we recorded. This is standard practice and you'll always use this system in your lab work. All you do is write down every digit you measure, including an estimated digit and you're done. These measured digits are referred to as **significant digits**. There is a bit of confusion involving zeros since a zero can indicate either a measured value (significant) or be placeholders (non-significant.) So let's look at how we deal with them.

Identifying significant digits

Rule 1. Non-zero digits are always significant.

Ex. 3.562 m – all four digits are significant.

Rule 2. Embedded zeros are always significant.

Ex. 2.05 cm – you measured the zero, so it's certainly significant. This measurement has three significant digits.

Rule 3. Leading zeros are never significant.

Ex. 0.0526 m – just three digits are significant.

Leading zeros are just place-holder digits. If you measured 0.0526 m in centimeters you'd write 5.26 cm. The units used should not change the number of significant digits. This number has three significant digits.

Rule 4. Trailing zeros are always significant. This is where it gets tricky. Look at the following three scenarios to see why.

Ex. 4.570 m – the zero comes to the right of a decimal point

You wouldn't include the zero unless you measured the last digit as a zero since 4.57 m is the same amount as 4.570 m. So you only needed to include the zero to indicate the uncertainty of the measurement.

Ex. 4570 m – all four digits are significant since you measured three and estimated the zero.

In this case you measured the 4, then the 5, then the 7, and then estimated that the last digit was zero. It's no different than, say, 4572. The last digit was the estimated digit and just happened to be zero.

Ex. 4570 m, **but** you measured the same object but with a different instrument with less precision.

So it's the same size measurement as the previous example, but the seven was the estimated digit. And the zero is just a placeholder. So you really have **only three significant digits**.

Well rules are rules. If that trailing zero is not significant, you have to get rid of it. Here's how. If you need some trailing zeros for placeholders, use scientific notation or metric prefixes instead.

Solution 1. $4.57 \times 10^3 \text{ m}$

Solution 2. 4.57 km

Significant figures in the result of a calculation

In addition to making measurements to the proper precision we also have to attend to significant digits when making calculations. There are basically two situations to be concerned with. Here's an example of each. Calculate what you think the answers are. We'll return to these questions after further study.

Ex. 1 Suppose you measured the inner diameter of a pipe to be 2.522 cm and the length to be 47.2 m. What would you list as its internal volume? _____ m³.

Ex. 2 A truck that weighs 32,000 pounds when empty is used to carry a satellite to the launch pad. If the satellite is known to weigh 2, 164.015 pounds what will the truck weigh when loaded? _____ pounds

Multiplication, division, roots and powers

After any of these operations, round the result off to the fewest significant digits possessed by any of the numbers in the calculation.

Ex. $2.53 \times 1.4 \rightarrow 3.542$ on the calculator

1.4 has the fewest significant digits. So we use 2 significant digits in our answer

Answer: 3.5

Addition and subtraction

Before adding or subtracting, round each number to the uncertainty of the least certain number. Then add or subtract and keep the common uncertainty.

Ex. $2.53 + 1.4 \rightarrow 2.5 + 1.4$

$$\begin{array}{rcl} 2.53 & \rightarrow & 2.5 \\ \underline{1.4} & \rightarrow & \underline{1.4} \\ & & 3.9 \end{array}$$

Notice that in the first case (\times , \div , etc.), we were counting sig. figs. In the second we were rounding to the least certainty. We won't discuss why this happens here. Just be aware of that key difference.

Now re-try example 1 above. Note that there's more than one calculation to make. Just do the rounding at the end of the calculation.

$$(V = \pi r^2 h = \pi (2.522/2 \text{ cm})^2 (4.72 \times 10^3 \text{ cm}) = 23578.8 \text{ cm}^2,$$

but we just get 3 digits so $V = 2.36 \times 10^4 \text{ cm}^2$

Now re-try example 2 above.

$$\begin{array}{rcl} (32,175 + 2,164.015) \text{ lb} = & 32,175 & \rightarrow 32,175 \\ & \underline{2,164.015} & \rightarrow \underline{2,164} \\ & & 34,339 \text{ lbs} \end{array}$$

Rounding

In both cases above we needed to round numbers, which involved removing trailing digits. There are different conventions for doing this. Here's how we'll do it.

Suppose we want to **round to n significant digits**.

Rule 1: If the digit immediately to the right of the n^{th} digit is **less than 5**, the number is rounded down.

Ex1. Rounding 2.434 to 2 significant digits becomes 2.4 after dropping 34.

Rule 2: If the digit immediately to the right of the n^{th} digit is **greater than 5**, the number is rounded up.

Ex2. Rounding 2.464 to 2 significant digits becomes 2.5 after dropping 64.

Rule 3: If the digit immediately to the right of the n^{th} digit is **5 and there are non-zero digits after the 5**, the number is rounded up.

Ex3. Rounding 2.454 to 2 significant digits becomes 2.5 after dropping 54.

Rule 4: If the digit immediately to the right of the n^{th} digit is **5 and there are no subsequent non-zero digits**, round the number in whichever direction leaves the n^{th} digit even.

Ex4a. Rounding 2.450 to 2 significant digits becomes 2.4 after dropping 50.

The result was left as 2.4 since 2.450 is closer to 2.4 than 2.6.

Ex4b. Rounding 2.550 to 2 significant digits becomes 2.6 after dropping 50.

The result was increased to 2.6 since 2.550 is closer to 2.6 than 2.4.

1.4 – Percent Error and Percent Difference

In the lab we will often try to determine the value of some “known” or theoretical quantity – the speed of light in a vacuum, the wavelength of light given off by an electrical discharge through a certain gas, or the acceleration of an object falling without friction. One way of **comparing our experimental result with the theoretical value** is calculating a percent error between the theoretical and experimental value.

The **error** between the theoretical and experimental values is just the absolute value of their difference.

$$\text{Error} = |\text{experimental} - \text{theoretical}|$$

We can get a better measure of the significance of our error by comparing it to the theoretical target value. We can do this by calculating a **percent error**.

$$\text{Percent Error} = \frac{|\text{experimental} - \text{theoretical}|}{|\text{theoretical}|} \times 100\%$$

Ex. The accepted value for the acceleration due to gravity is 9.82 m/s^2 at a certain location. In the lab a student measures it as 9.91 m/s^2 .

$$\text{Error} = |9.82 - 9.91| \text{ m/s}^2 = .09 \text{ m/s}^2$$

$$\text{Percent Error} = \frac{.09 \text{ m/s}^2}{9.82 \text{ m/s}^2} \times 100\% = .916\%$$

The error doesn’t mean much by itself. But by comparing it to the theoretical value of 9.82 we give it context. Calculating the percentage error tells us how big the error was relative to the amount of the measurement. A 1% error would mean that the student was off by 1 part in 100, so .916% is a bit less than that.

Sometimes we will want to compare two values for the same quantity when neither is known to be more certain than the other – the distance to a distant wall found with a meter stick and by sound reflection for example. In this case we can calculate a **percent difference**.

$$\text{Percent Difference} = \frac{|\text{measurement}_2 - \text{measurement}_1|}{|\text{average}|} \times 100\%$$

Ex. The speed of sound is determined in two different ways, once by measuring the time for a sound to bounce off a nearby building, and then by setting up standing waves in a tube. The two results are 318 m/s, and 340.6 m/s. While the second is measured to greater precision, we have no way of knowing how accurate either result is.

$$\text{Percent Difference} = \frac{|318 - 340.6|}{|\frac{318 + 340.6}{2}|} \times 100\% = 6.86\%$$

2.0 – Graphical Analysis

In sections 2.0 – 2.15 many examples of the use of Logger Pro will be illustrated. It is essential that you do every example as it's presented. You will be using all these techniques frequently this year, so it's crucial that you learn the process now.

Much of what we do in physics involves finding relations between quantities. These relations are best described using **mathematical models** – equations that relate the quantities. E.g. Speed = distance/time. Probably the most well-known and important such mathematical relation involves pizza. Years of study in the laboratory has determined that doubling the diameter of a circular pizza quadruples its area. The inability of consumers to unravel this relation, thus allowing them to understand the pricing, is surely the reason why pizza is generally sold in a circular rather than square shape. (Well, there is the difficulty of tossing a square pizza crust.) Consider this NCLB exam question:

Pizza A is 6" in diameter and costs \$4.67. Pizza B is 9" in diameter and costs \$6.21

- a) Pizza A is the better buy.
- b) Pizza B is the better buy.
- c) Both prices are equivalent.
- d) Pizza B is bigger so it's a better buy.
- e) It depends on the number of slices.

In my experience answer (e) is the universal answer. Let's get to the bottom of this.

The key question is "how is the area related to the diameter?" A mathematical model, an equation summarizing the relationship, is what we need. Here's how we'd go about it.

1. Go buy a bunch of pizzas. Check.
2. I've already done the heavy lifting for you. Here's the data as I took as I consumed the, err, "apparatus."

Pizza	Diameter (in)	Area (in ²)
1	2.11	3.14
2	3.94	12.6
3	5.82	28.3
4	9.15	63.6
5	10.02	78.5
6	12.25	113.1
7	13.84	154.0
8	16.01	201.1
9	20.14	314.2

By convention, the **independent variable** is listed on the left and the **dependent variable** is listed on the right. In this case, determining which is which is a bit iffy. In most cases it's clearer – the distance a car travels varies with time so time is the independent variable and the distance follows. Since we want to think about how the area depends on the diameter we'll consider the diameter to be the independent variable. In a **controlled experiment**, which is our goal, any other potential variables must be held constant.

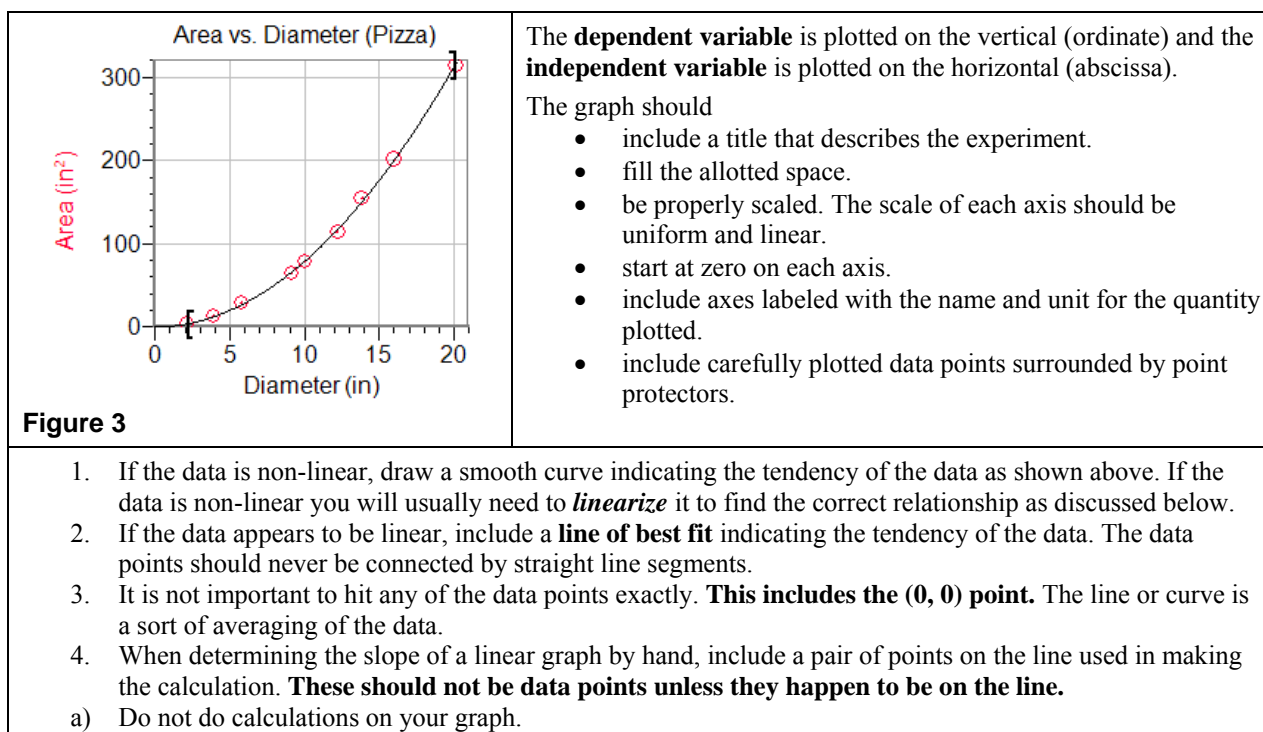
3. Make a table of area vs. diameter in Logger Pro. Here's how.

Open Logger Pro.
Double-click on the "X" column heading of the data table.
This will bring up the Labels and Units requester. Fill it in as shown. Click "Done."
Repeat for the "Y" column. Enter "Area", "A", and "in²".
To insert "in²", type "in", and then click the down arrow, choose "Superscript" and then "2."
Populate your table with the values in the table above.


Figure 2

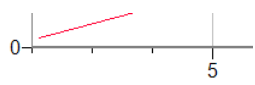
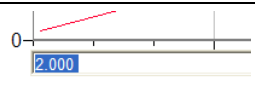

4. Plot area vs. diameter.


As you enter your data, Logger Pro will automatically plot it. The graph below is what we want for our finished product. Your graph shouldn't look much like this yet. Here's a description of what you want, followed by how to achieve it.



Here's how we'll achieve this finished look (with the exception of the curve.)

1. Once the data table is complete, double-click on the graph. Below "Title:" enter "Area vs. Diameter (Pizza)"
2. Click on the "Autoscale Graph" icon.  The data points will fill the space.
3. If either axis doesn't start at zero, click on the lowest number shown on that axis and change it to 0.

When I created this graph I found the lowest number shown on the x-axis to be 5 and that the x-axis actually started at 2.		Figure 4a
When I clicked 5, 2.000 appeared.		Figure 4b
When I changed the 2 to 0, I got the graph shown		Figure 4c

4. The axes will automatically match the data table. If you ever want to change these values, just change the data table as explained earlier.
 5. To see the data with point protectors, , and no connecting lines, double-click on the graph. Under "Appearance:" check "Point Protectors" and uncheck "Connect Points." You'll always want to do this.
 6. Your data is best summarized by your smooth curve or line of best fit. So once you create a line or curve, the data is no longer of any importance. To find the slope of a line by hand, pick two points on the line, as far apart as possible, and calculate the $\Delta y / \Delta x$. The slope should include units.
5. Determine an equation relating area and diameter.

This can be done in one of two ways using Logger Pro. **You should use the first method, linearizing the data in your labs.** You may find that Method 2, using Logger Pro's built-in curve fitting to be a good check of your results. It can also be used to draw your smooth non-linear curves.

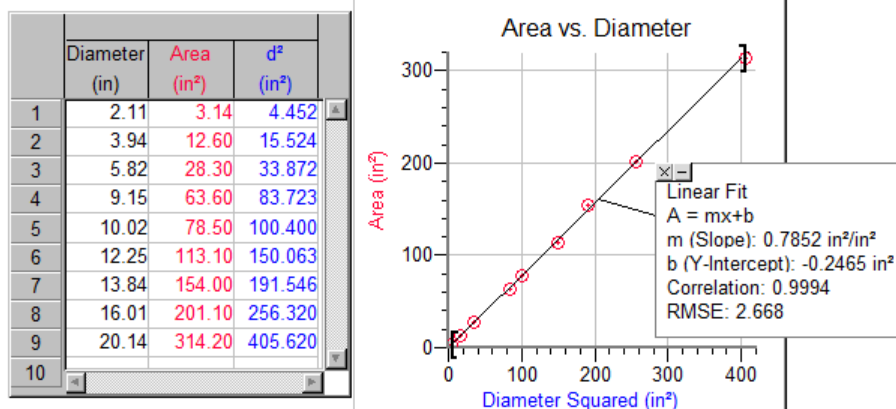




Figure 5

Method 1 (general description) – **Linearize data (Use this method for your lab reports.)**

- Create a new data table column. Have Logger Pro populate the column by raising the values in one of the initial columns to a power or root or some other function. In this example we've squared all the independent values and inserted them in the new column.
- Re-plot with this new data to see if a straight line is achieved. If not, repeat a.
- Select a linear region of the graph by dragging across it. See brackets [] on graph.
- Click the linear fit icon. 

Logger Pro will automatically calculate the slope and y-intercept of the **line of best fit** using **least-squares fit** calculation.
- Write the equation in $y = mx + b$ form.
y: ordinate value x: abscissa value m: slope b: y-intercept
- $A = (.78 d^2 + .39) \text{ in}^2$

Here's a **detailed explanation** of linearizing data with Logger Pro. You'll do this a lot in the lab, so pay attention to all the details.

- To create your new data column, click on "Data" in the menu at the top of Logger Pro. Select "New Calculated Column" from the list. Fill in the requester as follows.
Name: "Diameter Squared" Short Name: "D²" (again using down arrow for ².) Units: "in²"
You now want to populate this column with the square of each value in the diameter column.
Under "Equation:" in the same requester panel,
click the Variables button and select "Diameter". Note: The quotes are actually required here.
type "*" (no quotes), then select "Diameter" again. I don't know why there's no square function.
You should now have "Diameter"*"Diameter". Click "Done" and you'll have your new column. You may have to scroll to the right to see it or you can expand the data table area by dragging its border.
- You want to plot Area vs. Diameter Squared. Click the horizontal graph label, "Diameter (in)" and a list of other x-values appears. You should see Diameter, Area, Diameter Squared and More. Pick Diameter Squared. Autoscale and you'll have a nice straight line.
- This graph should all look pretty linear, so do something like this: Click anywhere inside the graph on the left side and drag to another point on the right side. A gray box should be created as you do this.
- Click the linear fit icon.  You should see something like the graph above.
- Write the equation in $y = mx + b$ form.
y: ordinate value (A) x: abscissa value(d) m: slope b: y-intercept
- $A = (.78(\text{in}^2/\text{in}^2) d^2 + .39) \text{ in}^2$ Not $y = (.78 x^2 + .39) \text{ in}^2$

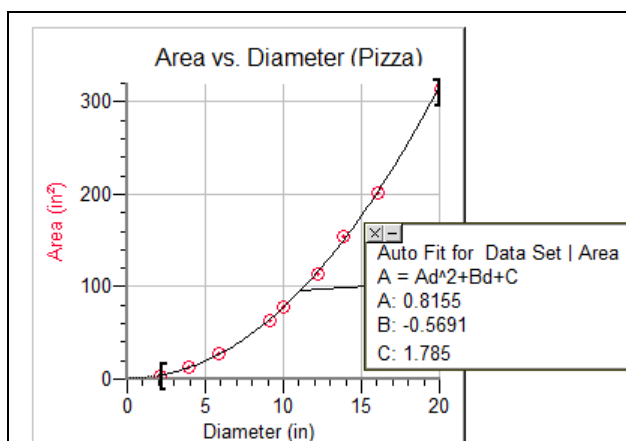



Figure 6

Method 2 – **Automatic curve fit** to a function. (You may use this to verify your equation from method 1.) Change the x-axis back to Diameter(in).

- Select a region of the graph by dragging across it. See brackets [].
- Click the curve fit icon. 
- Test the various general equations provided until you find one that produces the best curve to fit the data. Do this by selecting a general equation and then clicking “Try Fit.”
- Write the equation:

$$A = (.81 d^2 - .57 d - .39) \text{ in}^2$$

2.1 – Interpreting Graphs

In the pizza example we found this relation:

$$A = .78 d^2 + .39$$

Let’s look at the two constants m and b. Calculating $4 \times m$ gives you 3.12 which looks a lot like π . Recalling that the area and diameter of a circle are related by

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

explains where the .78 came from. It’s approximately $\pi/4$.

What about b, the y-intercept? Clearly a pizza of zero-diameter should have zero area. This tiny bit of pizza (.39 in²) is just a bit of random error. Random pizza? As we’ll see soon, this y-intercept will often have real physical meaning.

What we’ve found here is a common proportional relation – a quadratic proportion. While there are a huge number of possible relations, we’ll find only a handful in elementary physics. In some of our labs and course lectures we’ll be using a graphical approach to find or verify these relations, so let’s learn to recognize the ones that are likely to come up.

2.1.1 – No Relation

Don’t forget. You need to create all these graphs as you study this document. You’ll be making your own soon.

The time for a satellite to orbit a massive body is referred to as its period, T. This time depends on a number of factors. Surprisingly one of these is not the satellite’s mass. The following table illustrates what you might find for a satellite in low earth orbit. The period is not found to vary for satellites of different masses. Note that the correlation coefficient is 0. A perfect correlation would have a coefficient of 1.0. (See pizza graph.)

Although we do find a straight-line graph, the graph has no slope, indicating that the period does not change with mass. There is no relation between the mass and the period. Although there was no relation, we still had an independent variable, the mass, so it was plotted on the x-axis. What would the graph look like if we reversed these roles?

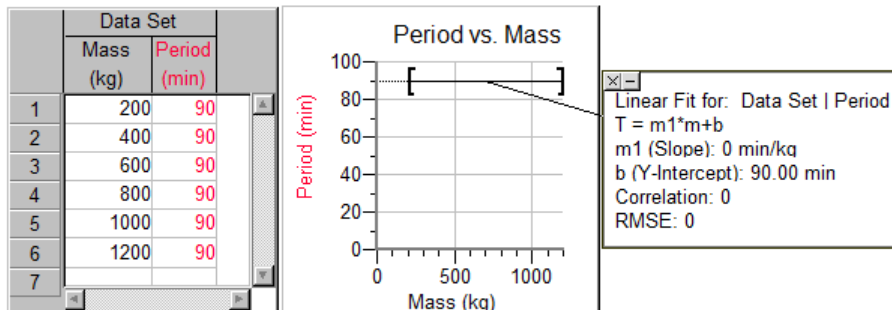


Figure 7

2.1.2 – Linear Relation with a y-intercept

An elastic object is one that stretches or compresses in a particular way (up to a point called the elastic limit) as increasing amounts of force are applied to it. A spring is a good example. A rubber band is not so good an example as its behavior changes as it's stretched by increasing amounts.

Consider a spring hanging vertically. If an object weighing one newton (about .225 pounds) is attached to the spring's lower end it will exert a downward force of one newton on the spring. The mass will vibrate for a while. Once it comes to rest, the spring will have a new length. The table below indicates this change in the spring's length as more weight is added.

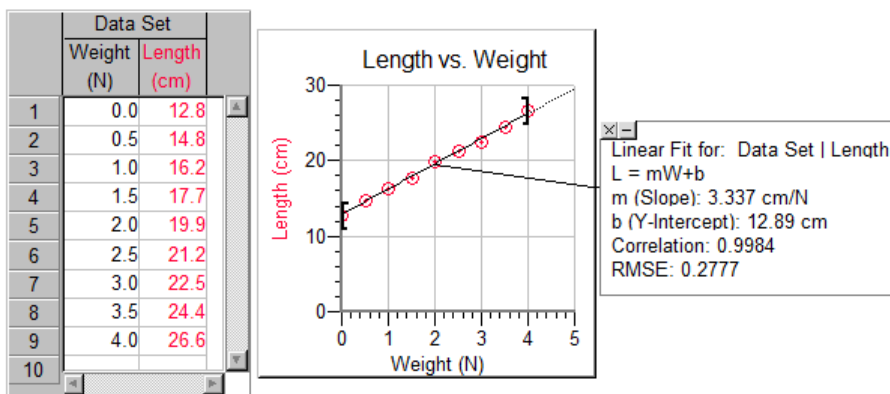


Figure 8

The spring is initially 12.8 cm long and increases steadily in length as weight is added. When we plot length vs. weight we get a straight line indicating a **linear relation** between length and weight. In a linear relation the dependent variable changes by a fixed amount for a fixed change in the independent variable.

Ex. A plot of fangs vs. snakes (for normally equipped snakes) would increase by 2 fangs for every snake added.

The graph indicates this relation by rising at a constant rate; that is, it has a **constant slope**. In our example, a slope of 3.337 cm/N indicates that the length of the spring increased by about 3.337 cm for each newton of weight added. You'd want to use 3.3 cm/N. Why?

Note that the slope can also be negative for a linear relation. For example an employee's salary vs. time might be defined by a rising straight line while the distance to her assigned parking space could decrease yearly resulting in a falling linear relation.

The y-intercept also has physical meaning. It's just the initial length of the spring before we began adding weights. Our equation that models this spring's stretchy behavior is

$$\text{Length} = 3.3 \frac{\text{cm}}{\text{N}} \text{Weight} + 12.9 \text{ cm}$$

Very nice! It sums up all the data in our graph plus everything outside it up to the elastic limit. A very tidy and efficient little package! If we kept adding weights until the spring began to distort, we'd find that the graph would start to bend toward the horizontal. So our equation works only within the spring's elastic range.

By the way, who cares?

This might be a time to discuss this burning question. Assuming that it's useful to be able to predict the length of a spring for example in designing cars, playground equipment, or whatever, what we've just done is really impressive. We've added some masses to the end of a spring and then gone from table \rightarrow graph \rightarrow equation. The equation is now something that we can carry around in our head that is a perfect model of the behavior of the spring. Think about it!

2.1.3 – Direct Proportion

If we rearrange the equation a bit we find an equation that gets more to the heart of the stretchy behavior of the spring. Subtracting the y-intercept, which is just the spring's initial length, from both sides gives us

$$\text{Length} - \text{Initial Length} = 3.3 \frac{\text{cm}}{\text{N}} \text{Weight}$$

or

$$\text{Stretch} = 3.3 \frac{\text{cm}}{\text{N}} \text{Weight}$$

To produce this equation directly from Logger Pro we can add a new calculated column labeled “Stretch” which is calculate from “Length” – 12.9 cm. Try it.

Notice how you can have two quantities plotted on the y-axis. Here's how it works. Click on the y-axis label “Length (cm).” Select “More.” Check the “Stretch” box.

Now a check box for each ordinate value will appear when you click the Linear Fit icon. Try it. You should get the result below.

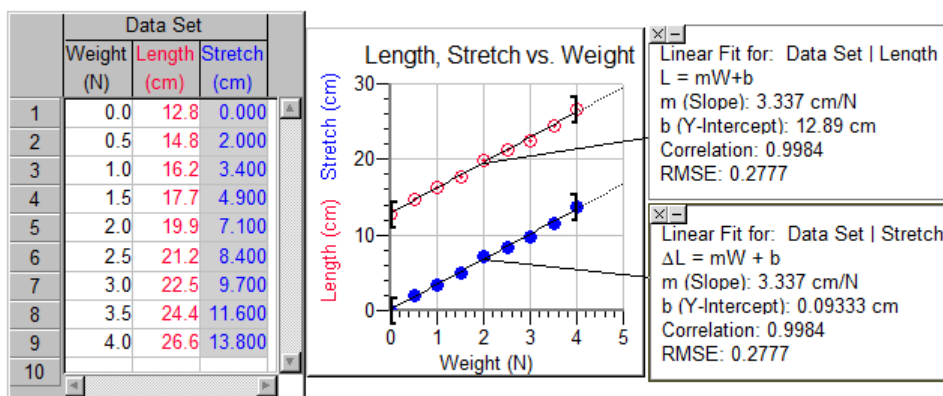


Figure 9

Since the stretch will be zero when the weight is zero our new line will (within the range of error) pass through the origin of our graph. Since it has a constant slope and passes through the origin, doubling the weight will double the stretch, tripling the weight will triple the stretch, etc. This relation is called a **direct proportion**.

Two quantities are **directly proportional** if they vary in such a way that one of the quantities is a constant multiple of the other, or equivalently if they have a constant ratio.

Why doesn't our data table exactly indicate this? First of all, our line of best fit doesn't pass exactly through the origin, but it comes close enough that we can attribute the slight miss to random error in measurement. Secondly, **we should never pay any attention to the data in our data table once we've drawn our graph**. If you count the moving fish in your fish pond three times and got 32, 36, and 37, your best estimate of their number would be 35, the average, not one of the specific counts. The line of best fit is a similar sort of average. The data table above gives 7.1 cm for the stretch with 2.0 N added. But the line actually goes a bit below the center of that data point, more like 6.8. And our equation suggests the same.

$$\text{Stretch} = 3.3 \frac{\text{cm}}{\text{N}} (2.0 \text{ N}) + .1 \text{ cm} = 6.7 \text{ cm}$$

Note the distinction between a linear relation and a direct proportion:

A direct proportion implies a linear relation.

A linear relation does **not** imply a direct proportion.

2.1.4 – Inverse Proportion

We found that in a direct proportion, multiplying the independent variable by a certain value will result in the dependent variable changing by the same factor. E.g., for a rectangle, Area = length \times width. Doubling the width will double the area. We also find the inverse of this relation – doubling one quantity will halve the other. Let's return to our pizza to find this inverse proportion.

From a given amount of dough you can create a pizza of almost any size, but as the size (area) increases, something has to give. That something is thickness.

$$volume = area \times thickness$$

Let's assume our pizza dough comes in balls of diameter 4.6 in. Their volume would then be

$$volume = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.3\text{ in})^3 = 51\text{ in}^3$$

For any circular pizza made with such a similar ball of dough,

$$51\text{ in}^3 = area \times thickness$$

A graph of area vs. thickness illustrates the nature of the inverse proportion. Since the product of the area and thickness is fixed, increasing one decreases the other by the same factor. That is, doubling the thickness halves the area.

Two variables are inversely proportional (or vary inversely) if one of the variables is directly proportional with the multiplicative inverse of the other, or equivalently if their product is a constant.

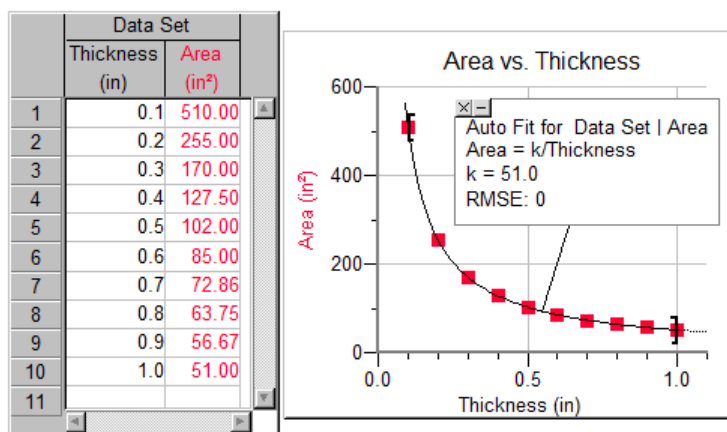


Figure 10

This relation can be stated as

$$k = x \times y \rightarrow x = \frac{k}{y} \rightarrow x \propto \frac{1}{y}$$

Just as with our initial pizza problem we can find the relation between the area and thickness by linearizing the data. First we create a new calculated column defined by 1/Thickness. You should be able to figure this out now. If you have trouble, be sure and ask for help.

This column is populated by the inverse of each thickness value. Plotting area vs. 1/T produces a straight line. So we can say that area is directly proportional to 1/T, the third statement above.

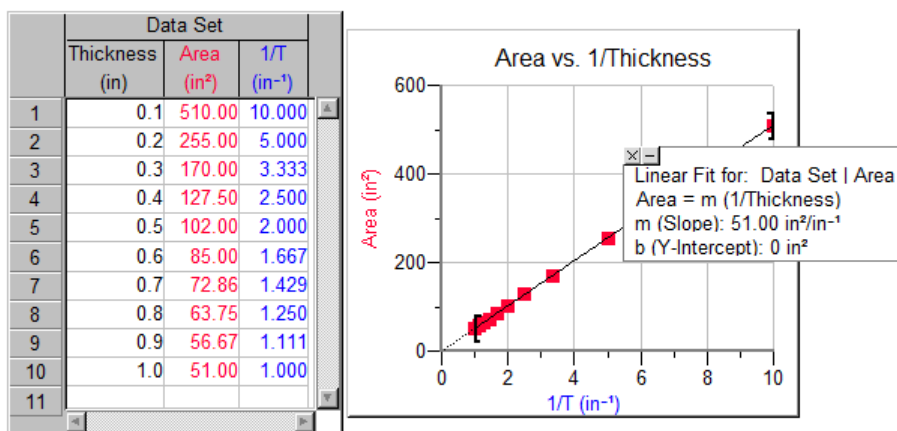


Figure 11

$$A \propto \frac{1}{T} \rightarrow A = \frac{k}{T} \rightarrow k = AT \rightarrow Volume = Area \times thickness$$

2.1.5 – Square or Quadratic Proportion

There are two different situations where we encounter quadratic proportions.

1) $y \propto x^2$ or $y = kx^2$	The independent variable is directly proportional to the square of the dependent variable.
2) $y^2 \propto x$ or $y^2 = kx$	The square of the independent variable is directly proportional to the dependent variable.

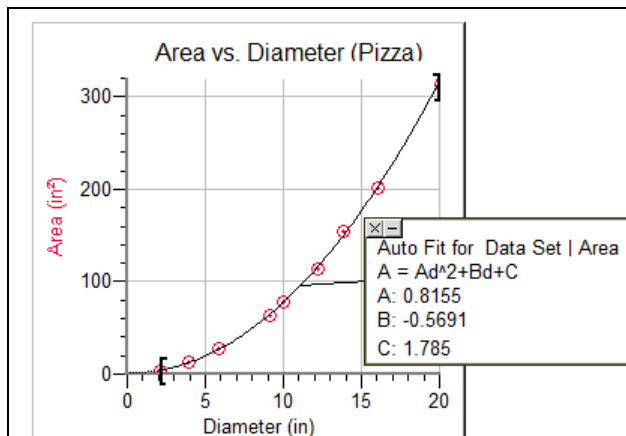


Figure 12

We've already seen the first one. By plotting Area vs. diameter squared we obtained a straight line with slope $.78 \text{ in}^2$ giving us the equation

$$A = .78 \text{ in}^2 d^2$$

(plus a y-intercept and constant that were small enough to be ignored.)

This is the equation of a parabola. More specifically it's an **upward opening parabola**. This is the nature of the graph of $y \propto x^2$.

The other type of quadratic proportion is exhibited by waves traveling in a string such as those in a guitar. When a guitar string is plucked waves travel quickly along the string and reflect back and forth between the fixed ends. The frequency of the sound produced is determined by the frequency of these round trips. The faster the waves travel the higher the frequency. The speed is adjusted by tightening and loosening the strings using the tuning pegs at the top of the guitar. Thus there is a relationship between the velocity of the waves and the tension in a string. Let's find it.

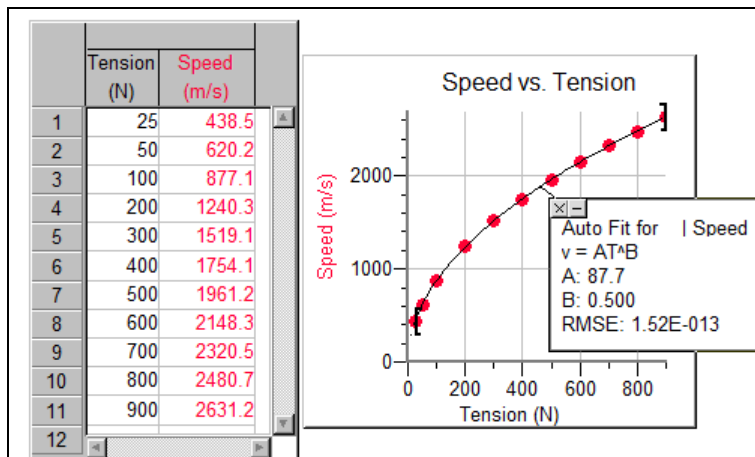


Figure 13

Here's our data for a certain string. Letting Logger Pro fit the curve (Method 2) we find that

$$v = 87.7 T^{.5}$$

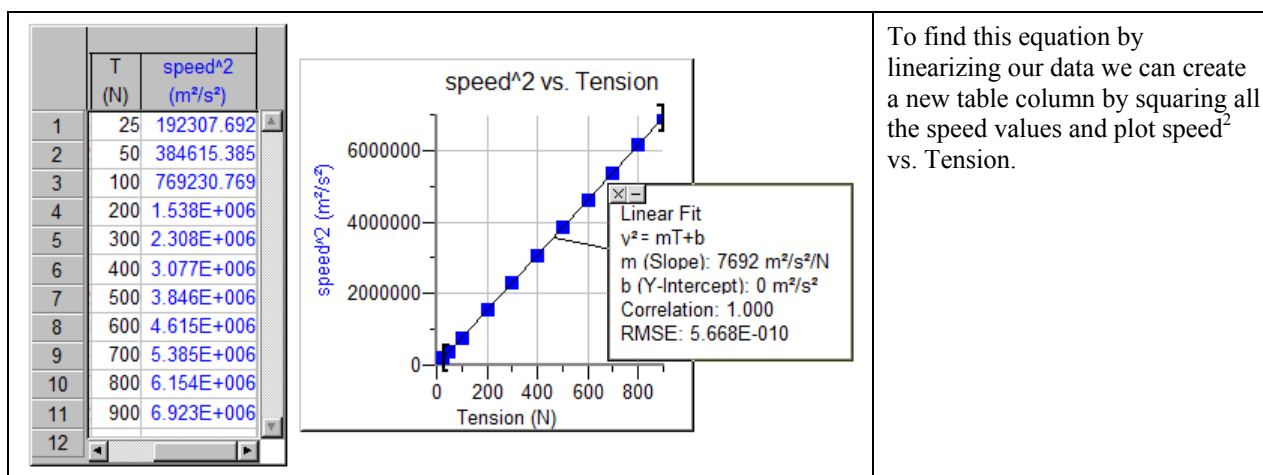
is the best fit.

To write this as a function of T we square both sides to produce

$$v^2 = 7692 T$$

which is in the form of the second quadratic relation

$$y^2 \propto x$$



To find this equation by linearizing our data we can create a new table column by squaring all the speed values and plot speed² vs. Tension.

This time we get the equation

$$v^2 = (7692 \text{ m}^2/\text{s}^2/\text{N}) T$$

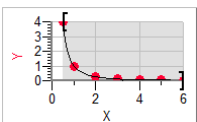
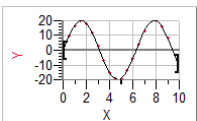
This is again the equation of a parabola. This time we have a **side opening parabola**. This is the nature of the graph of

$$y^2 \propto x.$$

Graphical Analysis Summary

In the lab we'll see relations 2-5 modeled. Relations 6-7 will be seen in class.

You'll want to refer back to this chart as you work with the data in your labs.

Relation between variables	Graph	Equation
1. No relation	Horizontal line	$y = \text{const.}$
2. Linear relation with y-intercept	Straight, non-horizontal line not through origin	$y = mx + b$ m: slope, b: y-intercept
3. Direct proportion	Straight, non-horizontal line through origin	$y = mx$ m: slope
4. Inverse proportion	Hyperbola	$y = 1/x$
5. Quadratic proportion	a. upward opening parabola b. side opening parabola	$y = kx^2$ $y^2 = kx$
6. Inverse square proportion		$y = k/x^2$
7. Sinusoidal		$y = A \sin(x)$ (shown) $y = A \cos(x)$ $y = A \tan(x)$